

Matching Large-Scale $p\text{CO}_2$ in Scale-Down Models: A Practical Manuscript

Abstract

Large-scale mammalian cell culture often runs at higher dissolved CO_2 partial pressure ($p\text{CO}_2$) than bench or pilot scale due to geometry-driven mass transfer and gas holdup differences. Failing to reproduce large-scale $p\text{CO}_2$ in a scale-down model (SDM) can bias pH control, buffer utilization, lactate/pyridine nucleotide metabolism, product quality, and CQAs. This manuscript unifies (i) a semi-empirical bicarbonate- CO_2 -lactate acid-base balance to compute the equilibrium dissolved CO_2 required at a given pH setpoint and buffer state and (ii) a headspace overlay blending model to set the overlay $\%\text{CO}_2$ (and/or flow) that attains a target gas-phase $p\text{CO}_2$ (e.g., 100 mmHg) observed at commercial scale. The result is an SDM recipe you can use directly in Excel or code to match large-scale $p\text{CO}_2$ while maintaining the intended pH and buffer equivalents.

1. Background and Problem Statement

At production scale, bioreactors frequently exhibit dissolved $p\text{CO}_2 \approx 80\text{--}120$ mmHg because of increased gas holdup, longer bubble residence time, deeper headspace, and different gas-liquid interfacial dynamics. In contrast, lab-scale vessels with high sweep and efficient stripping often stabilize around 40–60 mmHg unless enriched. When $p\text{CO}_2$ is not matched, SDMs underestimate acid load from carbonic acid, overestimate apparent base capacity, and may misrepresent lactate dynamics and glycosylation outcomes.

Goal: Provide a reproducible method to (1) compute the required dissolved CO_2 for a target pH given lactate and inorganic carbon pools, and (2) prescribe overlay $\%\text{CO}_2$ and flow to achieve a specified gas-phase $p\text{CO}_2$ (e.g., the 100 mmHg measured at large scale), even when the sparge remains air.

Assumptions

A) Core chemistry

- Fixed temperature = 37 °C: Constants are set for 37 °C ($\text{pK}'_a \approx 6.10$; Henry's constant $H_{37} \approx 0.0309$ mM/mmHg). Update them if temperature changes.
- Henderson–Hasselbalch applies to $\text{CO}_2/\text{HCO}_3^-$: pH is governed by carbonic acid vs. bicarbonate/carbonate. Accuracy drops with very strong non-carbonate buffers or rapid transients.

- Base equivalents only from bicarbonate/carbonate: We use $B_{eq} = [HCO_3^-] + 2[CO_3^{2-}]$. Other buffers (phosphate, amines) are ignored unless you extend the model.
- Lactate is an acid load: Lactate (g/L) is converted to mM on the anion basis (MW 89.07). If your assay reports lactic acid, use MW 90.08.

B) Linking liquid and gas CO₂

- Henry's law is linear at 37 °C: $dCO_2 = H_{37} \times pCO_2$. Media composition/foaming can shift effective solubility.
- Headspace pressure ≈ 1 atm unless specified: Gas fraction $y_{base} = \frac{pCO_{2,base}}{P_{tot}}$
use actual P_{tot} if reactor is pressurized or under vacuum.

C) Headspace overlay blending

- Steady state; CER cancels using baseline: We eliminate unknown CO₂ evolution by referencing the baseline, leading to

$$X_{ol} = y^* + \frac{Q_{sp}}{Q_{ol}}(y^* - y_{base})$$

- Well-mixed headspace: Overlay gas mixes before exhaust; poor mixing can cause off-gas lag vs. true headspace.
- Sparge is CO₂-free (air/air+O₂) in the basic model: If sparge contains CO₂, include it explicitly in the balance.

D) Scale-down fidelity

- Large-scale pCO_2^* is the target to reproduce: Use the measured plant headspace pCO_2 (e.g., 100 mmHg) as the benchmark; ensure probe placement/pressure are accurate.
- Mass transfer in SDM must be sufficient: If headspace pCO_2 matches but dCO_2 is low, increase overlay flow or %CO₂, or reduce total sweep (limit stripping).

E) Control & dosing simplifications

- Base top-up via bicarbonate/carbonate: Raising dCO_2 adds acid load; offset with NaHCO₃ (1 mEq/mM) or Na₂CO₃ (2 mEq/mM). Account for any strong-base pH control if present.

- Hold pH at the setpoint: All calculations assume the controller maintains the chosen pH (e.g., 7.10). Slow/saturated control will deviate.

2. Acid–Base Model (CO₂–Bicarbonate–Carbonate–Lactate)

We adapt the widely used semi-empirical formulation for bicarbonate-buffered media (37 °C). Let all concentrations be in **mM**.

2.1 Working balance

$$10^{(pH-pK_{a,CO_2})} = \frac{B_{eq} - L}{dCO_2} \Rightarrow dCO_{2,req} = \frac{B_{eq} - L}{10^{(pH-pK_{a,CO_2})}}$$

Where,

dCO_2 =dissolved CO₂ in liquid (mM)

L=Lactate (mM)

$B_{eq}=[HCO_3^-]+2[CO_3^{2-}]$ (mM);

$pK_{a,CO_2} \approx 6.10$ at 37 °C (tune ± 0.050 -0.10 depends on cell culture medium).

Interpretation. For a fixed pH setpoint, rising lactate increases acid equivalents and therefore increases the required dissolved CO₂ to satisfy (1) unless B_{eq} is augmented by bicarbonate/carbonate addition.

2.2 Lactate unit conversion (when reported in g/L)

$$L[\text{mM}] = \begin{cases} 1000 \cdot \frac{\text{lactate (g/L)}}{89.07} & \text{(lactate anion)} \\ 1000 \cdot \frac{\text{lactic acid (g/L)}}{90.08} & \text{(as acid)} \end{cases} \quad (2)$$

3. Linking Dissolved CO₂ to Gas-Phase CO₂

At 37 °C:

$$dCO_2 [\text{mM}] = H_{37} \left[\frac{\text{mM}}{\text{mmHg}} \right] \cdot pCO_2 [\text{mmHg}]; H_{37} \approx 0.0309 \quad (3)$$

$$pCO_2 [\text{mmHg}] \approx 7.6 \times \%CO_2 (\text{gas}) \quad (4)$$

Combining (3) and (4):

$$d\text{CO}_2 \text{ [mM]} \approx 0.23484 \times \% \text{CO}_2 \text{ (gas)} \Rightarrow \% \text{CO}_2 \approx \frac{d\text{CO}_2}{0.23484} \quad (5)$$

4. Headspace Overlay Blending to Match Large-Scale $p\text{CO}_2$

You observe: large scale $p\text{CO}_2^* = 100\text{mmHg}$; SDM baseline $p\text{CO}_{2,\text{base}} = 55\text{mmHg}$. Let:

- Q_{sp} = sparge flow (air),
- Q_{ol} = overlay flow,
- X_{ol} = overlay CO_2 fraction to be set,
- y^* is the target CO_2 fraction based on large scale. $y^* = p\text{CO}_2^*/760$
- y_{base} is simply the baseline mole fraction (or %) of CO_2 in the small scale.
We get it by converting the baseline CO_2 partial pressure (in mmHg) to a fraction of total pressure:

$$y_{\text{base}} = \frac{p\text{CO}_{2,\text{base}}}{P_{\text{tot}}}$$

- If the reactor headspace is at ~ 1 atm absolute, $P_{\text{tot}} \approx 760$ mmHg.
That's why you saw $y_{\text{base}} = p\text{CO}_{2,\text{base}}/760$.

Example: $p\text{CO}_{2,\text{base}} = 55$ mmHg \Rightarrow

$y_{\text{base}} = 55/760 = 0.0724 = \mathbf{7.24\% \text{ CO}_2}$ in the headspace.

Using a steady-state headspace material balance while eliminating unknown CER with the baseline condition yields a practical prescription:

$$X_{ol} = y^* + \frac{Q_{sp}}{Q_{ol}} (y^* - y_{\text{base}}) \quad (6)$$

Feasibility: If $X_{ol} > 1$, increase Q_{ol} , or increase the oxygen proportion in sparger to reduce Q_{sp} .

5. SDM Implementation Workflow

Inputs

- pH setpoint; measured/forecast lactate; bicarbonate and carbonate (mM).
- Large-scale target $p\text{CO}_2$ (e.g., 100 mmHg).
- SDM baseline $p\text{CO}_2$ (without CO_2 enrichment in the overlay) and gas flows Q_{sp} , Q_{ol} .

Step 1 — Acid–base requirement

Compute $B_{eq} = [\text{HCO}_3^-] + 2[\text{CO}_3^{2-}]$.

From (1), evaluate $d\text{CO}_2$, req for the chosen pH.

- If $B_{eq} - L \leq 0$, base is deficient; trim in bicarbonate/carbonate before $p\text{CO}_2$ matching.
- Otherwise, proceed.

Step 2 — Gas-phase target and expected dissolved CO_2

From the large-scale target $p\text{CO}_2^*$, compute

$$d\text{CO}_2^* = H_{37} \cdot p\text{CO}_2^* ; H_{37} \approx 0.0309 \text{ mmHg/ mM at } 37^\circ \text{C}$$

Example (at $p\text{CO}_2^* = 100 \text{ mmHg}$):

$$d\text{CO}_2^* = 0.0309 \times 100 = \mathbf{3.09 \text{ mM}}.$$

Check that $d\text{CO}_2^*$ and $d\text{CO}_2$, req are **compatible** with your pH and buffers. If not, adjust B_{eq} (bicarbonate/carbonate) or consider a slightly different $p\text{CO}_2$ target with justification.

Step 3 — Overlay CO_2 setpoint

Measure/estimate the SDM **baseline** headspace CO_2 fraction

$$y_{\text{base}} = \frac{p\text{CO}_{2,\text{base}}}{P_{\text{tot}}}.$$

With sparge flow Q_{sp} (air) and overlay flow Q_{ol} , set the required **overlay CO_2 fraction** X_{ol} by

$$X_{ol} = y^* + \frac{Q_{sp}}{Q_{ol}} (y^* - y_{base})$$

and convert to %CO₂ for the MFC: %CO₂^{overlay} = 100 X_{ol}.

(If X_{ol} > 1, increase Q_{ol}, decrease Q_{sp}, or add CO₂ to the sparge.)

6. Illustrative Example

Given

- pH = 7.10
- Lactate = 1.8 g/L
- Bicarbonate = 29 mM;
- Carbonate = 0 mM
- $pK'_a = 6.10$
- H₃₇ = 0.0309 mM/mmHg
- Large Scale Target pCO₂* = 100 mmHg
- *SDM baseline pCO₂ = 55 mmHg*
- Q_{sp} = 0.002vvm
- Q_{ol} = 0.02vvm

Step 1 — Check Acid–base requirement (unchanged)

$$R = 10^{(pH-pK_a)} = 10^{(7.10-6.10)} = 10$$

$$dCO_{2, req} = \frac{B - L}{R} = (29 - 20.2)/10 = \mathbf{0.88 \text{ m}} \quad \mathbf{M}$$

This just tells you: at pH 7.10, given B_{eq} and L, you'd “naturally” sit at **0.88 mM dCO₂**. Which is lower than the target of **3.09 mM**. **So Top of base is required.**

Step 2: Bicarbonate top-up Requirements ?

What is a base trim?

When you **raise $d\text{CO}_2$** (e.g., to match plant $p\text{CO}_2 = 100 \text{ mmHg} \rightarrow d\text{CO}_2^* \approx 3.09 \text{ mM}$), you add carbonic-acid load. To keep **pH 7.10** at the same lactate and buffer, you must add **extra base equivalents** (usually NaHCO_3 or a little Na_2CO_3). That addition is the **base trim**.

How much base?

Use the bicarbonate balance (37 °C):

$$d\text{CO}_2, \text{ req} = \frac{B_{\text{eq}} - L}{10^{(\text{pH} - \text{p}K'_a)}} \Rightarrow B_{\text{eq, needed}} = L + d\text{CO}_2^* \cdot 10^{(\text{pH} - \text{p}K'_a)}$$

Where $B_{\text{eq}} = [\text{HCO}_3^-] + 2[\text{CO}_3^{2-}]$.

Given

$\text{pH} = 7.10$, $\text{p}K'_a = 6.10 \Rightarrow 10^{(\text{pH} - \text{p}K'_a)} = 10$

Lactate $L = 20.2 \text{ mM}$

Current $B_{\text{eq}} = 29 \text{ mM}$ (29 mM HCO_3^- , no CO_3^{2-})

Target $d\text{CO}_2^* = 3.09 \text{ mM}$

Compute

$$B_{\text{eq, needed}} = 20.2 + (3.09 \times 10) = 51.1 \text{ mM}$$

$$\Delta B_{\text{eq}} = B_{\text{eq, needed}} - B_{\text{eq, current}}$$

$$= 51.1 - 29 = 22.1 \text{ mM}$$

Convert to a practical addition

- **As NaHCO_3** (1 mEq base per 1 mM): add **22.1 mM NaHCO_3**
→ grams/L: $22.1 \text{ mmol/L} \times 84.01 \text{ mg/mmol} = 1.8 \text{ g/L}$

- **As Na_2CO_3** (2 mEq base per 1 mM): so $22.1 \text{ mM}/2 = \mathbf{11.05 \text{ mM Na}_2\text{CO}_3}$
 \rightarrow grams/L (anhydrous): $11.05 \times 105.99 = 1.17 \text{ g/L}$

Step 3 — Overlay CO_2 setpoint

Large scale target: $p\text{CO}_2^* = 100 \text{ mmHg}$

$$\Rightarrow y^* = 100/760 = 0.1316$$

SDM baseline: $p\text{CO}_{2,\text{base}} = 55 \text{ mmHg}$

$$\Rightarrow y_{\text{base}} = 55/760 = 0.0724$$

SDM flows: $Q_{sp} = 0.002 \text{ vvm}$, $Q_{ol} = 0.02 \text{ vvm}$

$$\Rightarrow Q_{sp}/Q_{ol} = 0.1$$

Use

$$X_{ol} = y^* + \frac{Q_{sp}}{Q_{ol}} (y^* - y_{\text{base}})$$

Compute:

- $y^* - y_{\text{base}} = 0.1316 - 0.0724 = 0.0592$
- $\frac{Q_{sp}}{Q_{ol}} = 0.002/0.02 = 0.1$

So,

$$X_{ol} = 0.1316 + 0.1 \times 0.0592 = 0.1316 + 0.00592 = \mathbf{0.13752}$$

$$\Rightarrow \% \text{CO}_2 \text{ in overlay} = 100 \times X_{ol} \approx \mathbf{13.75\%}$$

(Feasible; well below 100%. If headspace/ $d\text{CO}_2$ still lag due to stripping, raise Q_{ol} slightly or nudge X_{ol} up.).

Appendix A

Rationale for omitting $k_L a$ from the SDM $p\text{CO}_2$ model.

Our objective is a **setpoint** problem: reproduce the large-scale headspace $p\text{CO}_2$ and maintain pH at the chosen setpoint. At steady state, the required quantities are governed by **gas composition and equilibrium** (overlay fraction/flows and Henry's law), not by transfer *rates*. Accordingly, we (i) prescribe the overlay % CO_2 to attain the target headspace fraction $y^* = p\text{CO}_2^*/P_{\text{tot}}$, and (ii) compute the **base equivalents** needed to hold pH given $d\text{CO}_2$, lactate, and bicarbonate/carbonate.

We **do not explicitly include** $k_L a$ for two practical reasons:

1. **Headspace setpoint is independent of $k_L a$.** The steady-state headspace composition results from inlet gas fractions and overlay/sparge flows. $k_L a$ influences the *rate* at which the liquid approaches equilibrium, not the headspace composition we set.
2. **Bench-scale transfer is not rate-limiting under our conditions.** In <5 L reactors operated here, $d\text{CO}_2$ closely tracks $H_{37} p\text{CO}_2$ at steady state (rapid step responses; $d\text{CO}_2^{\text{meas}}/H_{37} p\text{CO}_2 \approx 1$). Thus, **mass transfer is sufficiently fast** that CO_2 is not limiting for the steady-state setpoints we target.

As a safeguard, we **empirically verify** that measured $d\text{CO}_2$ matches the equilibrium value implied by the headspace. If stripping or dynamics cause a systematic shortfall, we correct operationally—**increasing overlay % CO_2 or Q_{ol}** , or reducing total sweep—without introducing an uncertain $k_L a$ parameter. We would include $k_L a$ only for **transient predictions, co-sparged CO_2** , or scenarios where **steady-state $d\text{CO}_2$ deviates materially from equilibrium**.